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ABSTRACT

According to the feature of gas flowing in small clearance, a mathematical model based on fluid mechanics and thermodynamics, is adopted for calculating leakage rate. Calculating and experimental results show that the computing method in this paper is better than that of nozzle flow and viscous flow.

NOMENCLATURE

A	area of leakage passage
D	equivalent diameter of leakage channel
E	eccentricity
f	partial friction coefficient
H	height of leakage passage
H_{min}	minimal height of leakage channel
K	specific heat ratio
M	March Number
P	pressure
P_h	high pressure
P_l	low pressure
R_c	cyclinder radius
R_e	Renold Number
r	piston radius
S	section area of leakage passage
T	temperature
T_h	temperature of high pressure gas
U	gas flowing velocity
W	width of leakage passage
dA_w	element area of leakage passage
ρ	density
ν	viscosity
θ	angle

INTRODUCTION

Gas leakage is one of the main factors affecting the performance of compressor. It would be done better to predict the performances of compressor if the leakage rate is computed more accurately. For calculating the leakage rate in compressor, most literatures [1,2] suggest to use nozzle flow model, in which the effect of viscous friction on flow is neglected. In other literature [3], viscous flow model is used, which neglectes the effect of inertia friction. In fact, as gas is ease to be compressed and it leaks through small clearance, both viscous force and inertia force should be considered in setting up mathematical model. In this paper, a compute model is derived. The calculating and experiment results show that the computing method in this paper is better than that of nozzle flow and viscous flow.

CALCULATION MODEL ESTABLISHMENT

In most case, the leaking passage is constructed by two curved surfaces (or planes) with same width, the change of section area is small. According to the rule that flowing section area is equal, the leaking passage could be transformed into the channel shown in fig. 1. The high pressure gas flows into passage through section 1-1, then passes the small channel in which section area is changing, finally leaves passage through section 2-2. It is assumed that flowing process is one dimensional, steady, frictional and heat insulation. Analysing any one elementary volume in flowing channel, we could obtain the following basical equation to describe the leakage flow

continuity eq. $$\frac{d\rho}{\rho}+\frac{dU}{U}+\frac{dA}{A}=0 \tag{1}$$

momentum eq. $$UdU+\frac{dP}{\rho}+\frac{P}{\rho}\frac{dA}{\rho}+\frac{\tau_w dA_w}{\rho A}=0 \tag{2}$$

state eq. $$\frac{dP}{P}=\frac{dT}{T}+\frac{d\rho}{\rho} \tag{3}$$

process eq. $$\frac{dP}{P}=K\frac{d\rho}{\rho} \tag{4}$$

where, friction force $F_w=\tau_w dA_w=\tau_w S_w dx$

friction stress $\tau_w=\rho U^2 f/2$

friction coefficient $f=\begin{cases} 96/R_e & R_e<3560 \\ 0.3164/R_e & R_e>3560 \end{cases}$

Reynold Number $R_e=UD/v$

Removing $d\rho/\rho$ from quations (1) and (2)

$$\frac{dU}{U}=-\frac{dP}{KP}-\frac{dA}{A} \tag{5}$$

Substituding the expressions of τ_w, F_w and dU for them in eq.(2), we could get

$$\frac{dP}{\rho}=\frac{KM^2-1}{1-M^2}\frac{dA}{A}-\frac{KM^2}{1-M^2}\frac{fSdx}{2A} \tag{6}$$

Combining eq.(4) and (6)

$$\frac{d\rho}{\rho}=\frac{KM^2-1}{K(1-M^2)}\frac{dA}{A}-\frac{M^2}{1-M^2}\frac{fSdx}{2A} \tag{7}$$

Replacing $d\rho/\rho$ in eq.(1) by above expression

$$\frac{dU}{U}=-\frac{K-1}{K(1-M^2)}\frac{dA}{A}+\frac{M^2}{1-M^2}\frac{fSdx}{2A} \tag{8}$$

By using eq.(3), (6) and (7), the following equation is derived

$$\frac{dT}{T}=\frac{K-1}{K}\left(\frac{KM^2-1}{1-M^2}\frac{dA}{A}-\frac{KM^2}{1-M^2}\frac{fSdx}{2A}\right) \tag{9}$$

According to $M=U/a$ and $a=\sqrt{KRT}$, we get $M^2=U^2/(KRT)$, expressing it in differential form

$$\frac{dM}{M}=\frac{dU}{U}-\frac{dT}{2T} \tag{10}$$

Substituding dU/U and dT/T in eq.(10) by them in eq.(8) and (9), the following equation can be obtained

$$\frac{dM}{M}=-\frac{(K-1)(KM^2+1)}{2K(1-M^2)}\frac{dA}{A}+\frac{K+1}{2}\frac{M^2}{1-M^2}\frac{fSdx}{2A} \tag{11}$$

Basied on above deducing, the general compute model is expressed as

$$\frac{dP}{P}=\frac{1}{1-M^2}\left((KM^2-1)\frac{dA}{A}-KM^2\frac{fSdx}{2A}\right) \tag{12}$$

$$\frac{dT}{T}=\frac{K-1}{K(1-M^2)}\left((KM^2-1)\frac{dA}{A}-KM^2\frac{fSdx}{2A}\right) \tag{13}$$

$$\frac{dU}{U}=-\frac{1}{1-M^2}\left(\frac{K-1}{K}\frac{dA}{A}-M^2\frac{fSdx}{2A}\right) \tag{14}$$

$$\frac{dM}{M}=-\frac{1}{2(1-M^2)}\left(\frac{K-1}{K}(KM^2+1)\frac{dA}{A}+(K+1)M^2\frac{fSdx}{2A}\right) \tag{15}$$

THE METHOD OF COMPUTE LEAKAGE RATE

While calculating leakage rate, the standard Runge-Kutta method is adoptd. The steps are as following

1. dividing the passage into N-1 small districts along the flow direction, every boundary line represents one section.

2. gaving T_o, P_o and U_o on initial cross-section, then computing T_i, P_i, U_i and M_i on section i by using Runger-Kutta method, where i=2, 3, ..., N successively.

3. checking the parameters on section N, if either of the following conditions is satisfied, the flowing field calculation is finished

(1) $|P_N - P_L| < \varepsilon$;
(2) choke flow .

4. correcting inlet velocity, if the condition is not satisfied

$$U_o = U_o + \Delta U$$

repeating above calculation until condition (1) or (2) is met.

5. computing the leakage rate and analysising the flowing field

(1) compute the leakage rate: according to the temperature, pressure, velocity and area on any section, the leakage rates could be calculated

$$Q = U_i A_i \quad \text{and} \quad M = U_i A_i \rho_i$$

(2) anaiysis the flowing field: draw the diagram of P-x and U-x, we can know the change regulars of pressure and velocity on every cross-sections in flowing field.

CALCULATION EXAMPLE AND DISCUSSION

Fig. 2 gaves a leaking passage constructed by two eccentric cylinder surface. The relationship between channel highness H and angle θ is

$$H(\theta) = R_c - (E - H_{min})\cos\theta - \sqrt{r^2 - (E - H_{min})^2 \sin^2\theta}$$

The element length

$$dx = \left[(E - H_{min})\cos\theta + \sqrt{r^2 - (E - H_{min})^2 \sin^2\theta}\right] d\theta$$

The passage width is W. So the wedded periment is $S=2(H+W)$, the section area is $A=HW$, and the equivalent diameter is $D=2HW/(H+W)$.

Because the passage is very thin, the maximum velocity is the speed of sound. $R_e <$ 3560, the partial friction coefficient is

$$f = 96/R_e = 96\nu/(UD)$$

Substituding the expressions of $H(\theta)$, dx, D and f for them in eq.(12), (13) and (14), we could get the following computing model

$$\frac{dP}{P} = \frac{1}{1-M^2}\left((KM^2-1)\frac{dH}{H} - KM^2\frac{fSdx}{2A}\right) \tag{16}$$

$$\frac{dT}{T} = \frac{K-1}{K(1-M^2)}\left((KM^2-1)\frac{dH}{H} - KM^2\frac{fSdx}{2A}\right) \tag{17}$$

$$\frac{dU}{U} = -\frac{1}{1-M^2}\left(\frac{K-1}{K}\frac{dH}{H} - M^2\frac{fSdx}{2A}\right) \tag{18}$$

$$\frac{dM}{M} = -\frac{1}{2(1-M^2)}\left(\frac{K-1}{K}(KM^2+1)\frac{dH}{H} + (K+1)M^2\frac{fSdx}{2A}\right) \tag{19}$$

where, $$\frac{fSdx}{2A} = \frac{48\nu}{U}\left(\frac{H+W}{HW}\right)^2\left[(E-H_{min})\cos\theta + \sqrt{r^2-(E-H_{min})^2\sin^2\theta}\,\right]d\theta$$

Fig.3 shows the relationship between pressure P and angle θ in flowing field. Obviously, no matter how the difference of pressure is, the change of presure yields in a small district which is close to the minimal clearance. While the difference between high pressure and low pressure is smaller, the pressure difference in flowing field is equal to that of outside. On the contrary, as the difference between high pressure and low pressure is larger, choke flowing occurs, the pressure difference in flowing field is only one part of outside pressure difference. Another part exists in low pressure chamber and causes the pressure pulse in low pressure chamber.

Fig.4 illustrates the relationship of leakage rate Q with pressure difference ΔP. Fig.5 shows the relationship between Q and minimal gap H_{min}. Curves a, b, c represent the compute results of nozzle model, viscous model and the model of this paper respectively. We could find that curve c is most colse to experiment results and it appears non-linear if the pressure differences and gaps are smaller. The reason is that the friction force plays an important rule in these conditions. On the contrary, when the pressure differences and gaps are larger, the curves are near to linear because the inertia force is dominant factor effecting flow.

CONCLUSIONS

1. In this paper, a compute model is derived. The calculating results are well agreement with the experimental results in literature [4].
2. The pressure drop in leakage passage occures in a small district which is close

to the minimal clearance.

3. The choke flow appears as the outside pressure difference is larger, the pressure pulse yields in low pressure chamber.

4. In compressor, since the leakage fluid is gas-oil mixture and the boundary moving direction is opposite to that of leak flowing, the practice leakage rate is smaller than the computing results.

REFERENCES

1. J.L.Caillat, A Computer Model for Scroll Compressor, Proc. of the 1988 Purdue International Compressor Engineering Conference.
2. Kenji Tojo, Computer Modeling of Scroll Compressor with Self-adjusting Pressure Mechanism, Proc.of the 1986 Purdue International Compressor Engineering Conference.
3. Yu Y.Z, Piston Type Compressor, Mechanary Industry Press, 1978.
4. Yuan Xiuling, Calculating Model and Experimental Investigation of Gas Leakage, Proc. of the 1992 Purdue International Compressor Engineering Conference.

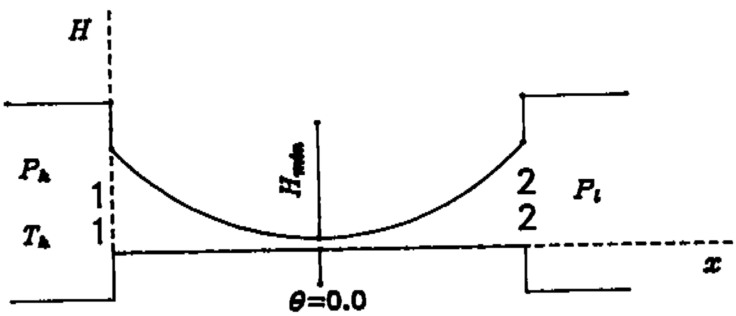

Fig.1 TRANSFORMED PASSAGE

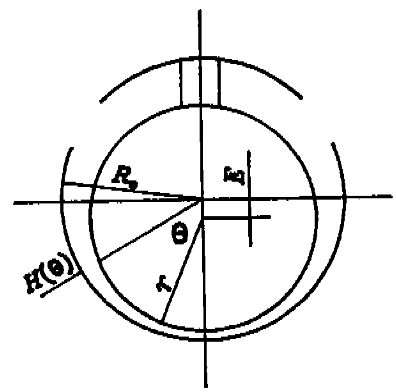

Fig.2 LEAKAGE PASSAGE

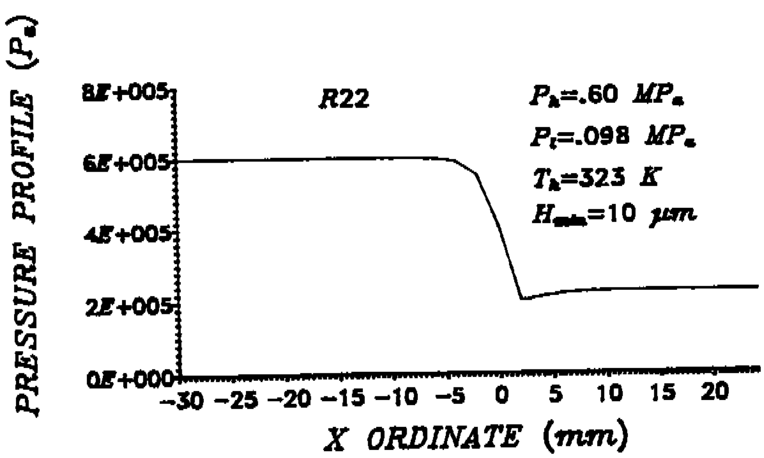

Fig.3 PRESSURE PROFILE CURVE

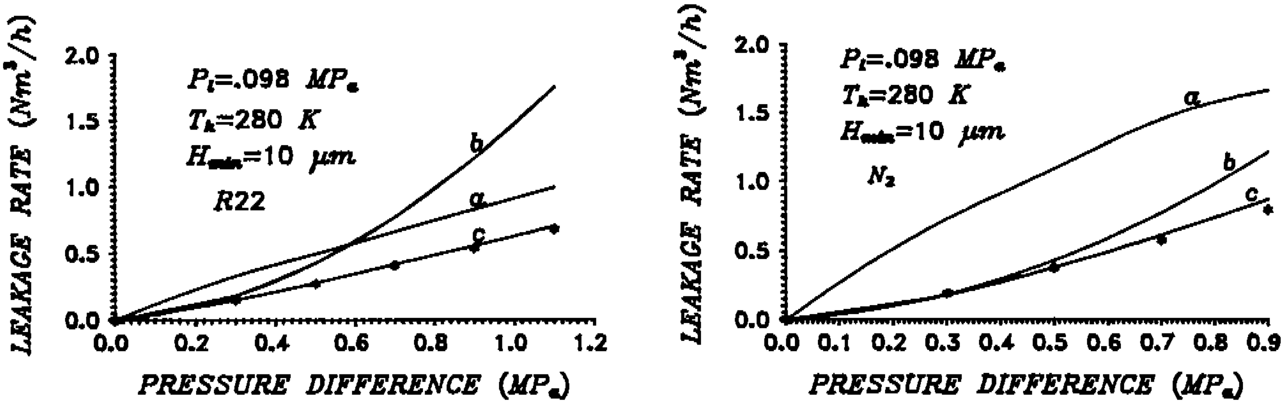

Fig.4 LEAKAGE RATE & PRESSURE DIFFERENCE CURVE

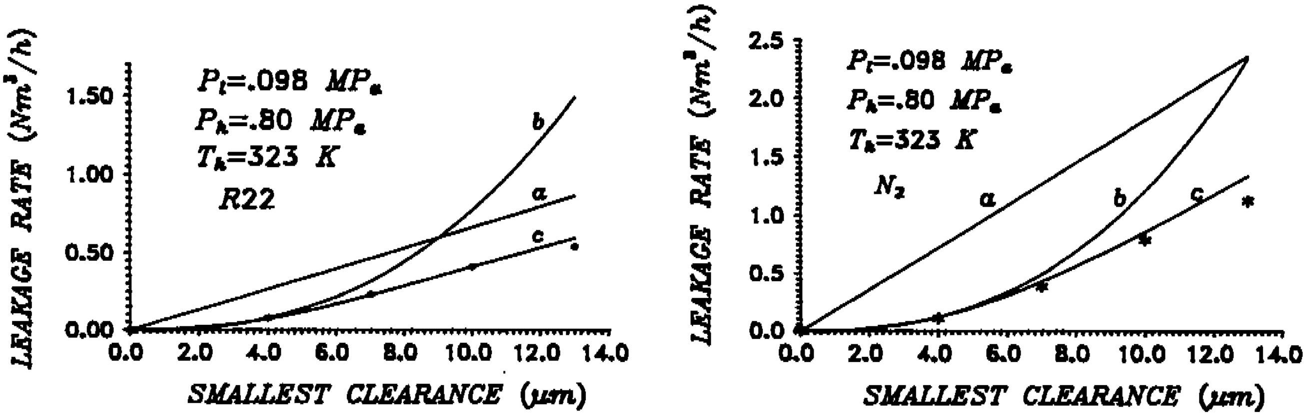

Fig.5 LEAKAGE RATE & SMALLEST CLEARANCE CURVE